THE GENERIC RIGIDITY OF MINIMAL CYCLES

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If Γ is an abelian group and X is a finite abstract simplicial complex, a p-cycle of oriented p-simplices of X with coefficients in Γ can be regarded as a formal sum that is mapped to 0 by the usual boundary operator of algebraic topology. If c and c' are p-cycles we say that c' is a subcycle of c if for every simplex σ the coefficient of σ in c' is either 0 or equals the coefficient of σ in c. We say c is minimal if its only subcycles are 0 and c. We call a finite abstract simplicial complex X a p-cycle complex if there is a group Γ and a p-cycle c of oriented p-simplices of X with coefficients in Γ such that X equals the support of c. We call X minimal if c is minimal. For $d \geq 3$, we show that a realization in d-space of the 1-skeleton of a minimal (d-1)-cycle complex is a generically rigid framework, i.e. the 1-skeleton is a generically d-rigid graph.

The proof is by induction on the number of vertices of X. We show there is a finite set of subgraphs of the 1-skeleton of X satisfying the following conditions: (1) each subgraph has an edge contained in at least d-1 triangles and such that when the edge is contracted a smaller graph is formed which is the 1-skeleton of another minimal (d-1)-cycle complex, or else the original subgraph is the 1-skeleton of a d-simplex; (2) if the set of subgraphs is partitioned into two subsets then the

union of the subgraphs in one subset shares at least d vertices with the union of the subgraphs in the other subset; (3) the union of the subgraphs equals the 1-skeleton of X. The 1-skeletons of d-simplices are generically d-rigid, and by a recent theorem of Walter Whiteley's, if a graph has an edge contained in at least d-1 triangles and such that when the edge is contracted a smaller graph is formed which is generically d-rigid, then the original graph is also generically d-rigid. Thus assuming that the 1-skeletons of minimal (d-1)-cycles with fewer vertices than X are generically d-rigid, condition (1) insures that the subgraphs are generically d-rigid. We show that condition (2) insures that their union is generically d-rigid, so that by condition (3) X is generically d-rigid. A corollary is that connected closed triangulated polyhedral surfaces are generically rigid.

Biographical Sketch

Allen Fogelsanger was born in Pennsylvania in 1962. He attended the Pennsylvania State University and graduated summa cum laude in 1983, receiving the degree of Bachelor of Science in Mathematics with Honors in Mathematics. He was awarded the degree of Master of Arts by Cornell University in 1986.

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